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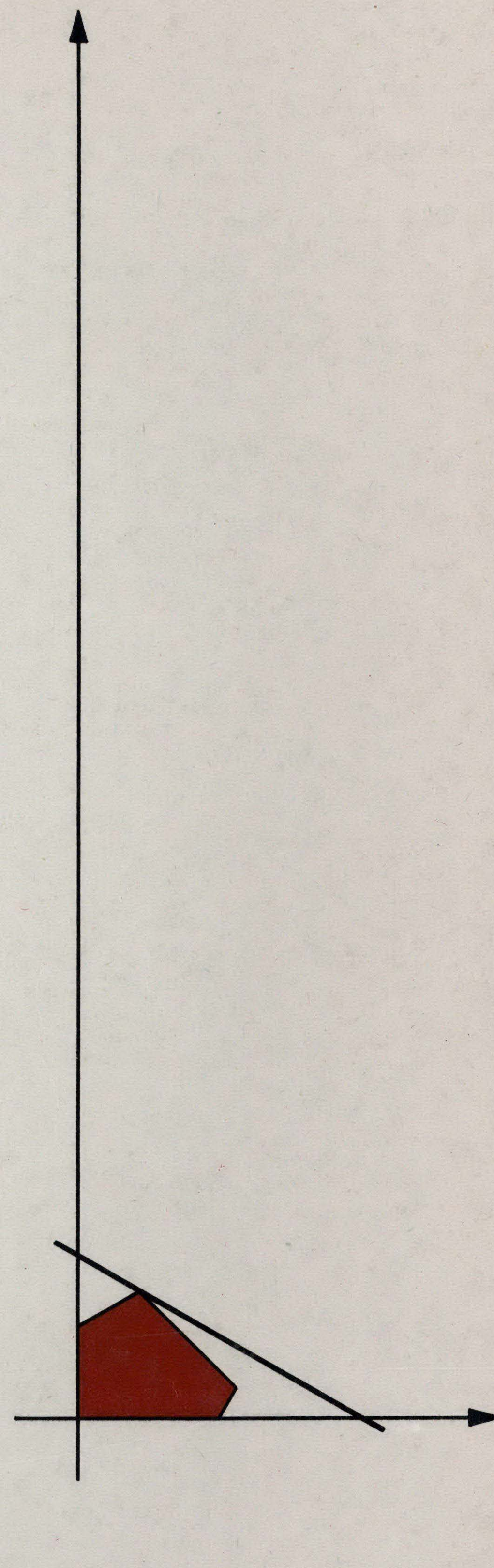

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PRELIMINARY ANALYSIS OF THE INFLUENCE OF HAND CREWS ON FIRE GROWTH

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ABSTRACT

Current procedures for suppressing forest fires include the use of hand crews as one of the major types of suppression forces. The effectiveness of hand crews is influenced by tactics, crew size, and fatigue. To examine these influences two tactical alternatives of assigning crews to a fire for both single and multiple shift situations are formulated and applied to three simple fire-spread models. Using a decision criterion based on minimizing costs, a procedure is suggested for finding the best combination of number of shifts and crew size to insure completion of a certain length of line in a certain period of time for a specified set of conditions.

PRELIMINARY ANALYSIS OF THE INFLUENCE OF HAND CREWS ON FIRE GROWTH

I. Introduction

Current firefighting procedures often require crews of men using hand tools to supply a major portion of the suppression effort on a fire since under certain conditions these crews may be the only suppression force which can be effective against the fire. Although the effects of their suppression effort may be considerably less than that of modern mechanized suppression equipment, the crews of men are often much more flexible.

Crews of men can be used in terrain in which it may be difficult or impossible for bulldozers to operate. Men can be dropped from airplanes and helicopters into remote areas which are inaccessible to bulldozers and ground tankers. They can work under low visibility conditions which would make the use of aerial tankers impractical. Often men are available to attack a fire before mechanized equipment can be brought into action. In some instances the quantity of mechanized equipment available may be limited and manpower may be required to complete the suppression effort. In certain phases of the suppression activities it may be cheaper but just as effective to use men as to use mechanized equipment.

Since hand crews are definitely one of the major types of suppression forces, it is appropriate in the early stages of an operational analysis of fire control systems to investigate their influence on fire growth.

Parks and Jewell (2) in their preliminary initial attack model indicated gross procedures for determining the number of men to send to a fire to insure control. One parameter of their decision model described in a gross way the effectiveness of the men in controlling the fire. This effectiveness "factor" was evaluated from historical data and represented an average over the time from initial attack until control was effected. No attempt was made in their study to examine how the actual deployment of the men on the fire might influence the value of their effectiveness.

Since, in reality, the major influence that a crew of men has on a fire is related to their ability to construct and "hold" control lines to hinder spread, the following analysis will examine in a preliminary way the behavior and effectiveness of hand crews on the fireline.

II. Manning Models

A. Basic Assumptions.--For a given set of conditions, such as fuel type and distribution, topography, training and physical condition of the men, etc., a crew may be expected to construct line at some rate which is a function of the length of time on the line and the number of men in the crew (1, 3). The U. S. Forest Service (3) presents data which is used to predict the decrease in production rates with time due to fatigue for a wide range of fuel types. These data were obtained from studies made of crews constructing line, presumably

using the "one lick method,"* under standardized working conditions, and working only one or two hours. The studies indicated that a man can only work with 90 percent of the first hour's effectiveness during the second hour of line construction, even after a 10 minute rest period between the two hours. The Forest Service extrapolated from these results for the third, fourth, and following hours. The resulting expression is approximately the following:

(1) $$R(t) = 0.833\ K(0.9)^{t-1}$$

where $R(t)$ = average rate of line construction during the t^{th} hour, distance/man-hour

K = rate of line construction during the first hour with no rest period allowance, distance/hour.

The factor 0.833 takes into account a ten minute rest period at the end of each hour. The value of the K factor of equation (1) is presented in reference 3 for different fuel types and is referred to as the "basic rate" for a given fuel type. These values, however, apply only for the average well-trained crew in good physical condition working on level or nearly level ground under comfortable conditions during daylight hours.

Matthews (1) presents data which shows the effect of crew size on line production and points out that this effect was due mainly to the amount of walking each man had to do to "get into position" on the line. Although the crews used line construction procedures which were

*The "one lick method" is the line construction procedure in which each man does one to several licks or strokes of work and moves forward a specific distance. The distance is determined by the number of men equipped with a given tool and the numberof licks needed per unit of line to complete the work for that tool.

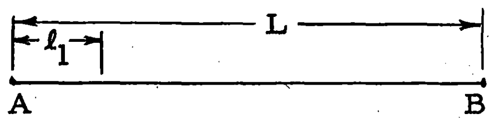
popular before the introduction of the "one-lick method," it seems reasonable to assume that the effect of crew size observed by Matthews will also hold for the "one-lick method." Fatigue was observed to decrease line production but no data was given.

Therefore, in an attempt to maintain consistency and simplicity in the initial manning models to be presented below, the following assumptions are made:

1. The term "man" can mean either an individual man or an individual crew of a fixed size.
2. The number of men or the number of crews assigned to a certain length of line influences the resulting line production only when the men or crews must walk to their positions on the line.
3. The "holding" aspect of control line construction may be ignored since it is possible to provide holding crews to follow the construction crews.
4. The average rate of line construction is a function of time spent on the line only.

B. Manning Model No. 1.--This model assumes that a crew of X men has been deposited as a group at the spot which is the starting point of the length of line L to be constructed by that crew during the time interval T_L. All of the men in the crew are to arrive at the end of the length of line assigned to the crew at the end of the time interval T_L at the same time so that they may all be transported back to camp as a group.

In this particular model the first man starts constructing line immediately at point A.



He finishes his length of line, ℓ_1, in time to walk the distance $L - \ell_1$ to point B by the end of the time period. The second man walks the distance ℓ_1 to the point where he starts building line. He builds line of length ℓ_2 and then walks the distance $L - \ell_1 - \ell_2$ to point B by the end of the time period. This process is continued for the rest of the men in the crew with the Xth man walking the distance $L - \ell_X$ and building line of length ℓ_X so that he just completes his line construction at point B at the end of the time period.

The main characteristic of this model is that each man works the same length of line, $\ell = L/X$, in the same length of time t_ℓ if a constant walking rate W is assumed which is equal for all men in the crew. Each man also walks the same length of line $L - \ell$. The relationship between L, T_L, and X is given by

(2)
$$L = X \int_0^{T_L - L/W \frac{X-1}{X}} R(t)dt$$

where R(t) is some mathematical expression, such as equation (1), for the average rate of line construction by one man at time t after he starts work. The derivation of equation (2) is presented in the appendix.

This manning model essentially represents the situation which results from use of the "one lick method." The grouping of the licks that each man performs into an average total length of line, ℓ, simplifies the visualization of the method.

C. Manning Model No. 2.--In this model the goal is to build a total length of line L in a given time T_L as for Model No. 1. However, although all of the men in the crew start out from point A at the same time, the first man constructs line during the whole time period T_L. The second man walks the distance ℓ_1 and then starts building line of length ℓ_2 and continues line construction until the end of the time period. Therefore the j^{th} member of the crew walks a distance of

$$\sum_{i=0}^{j-1} \ell_i \quad , \text{ where } \ell_0 = 0 \, ,$$

and constructs line until the end of the time period. The walking rate W is assumed constant and equal for all men in the crew. This model requires that only the X^{th} man be at point B at the end of the time period.

The relationship between L, T_L, and X for this model is given by

$$L = \sum_{j=1}^{X} \int_{0}^{T_L - \sum_{i=0}^{j=1} \frac{\ell_i}{W}} R(t)dt \tag{3}$$

where $R(t)$ has the same meaning as in equation (2) and $\ell_0 = 0$.

This manning model might occur in practice when an initial attack crew is sent out to get a certain length of line constructed as quickly as possible.

D. Specific Forms of $R(t)$.--Equations (2) and (3) imply for all but the simplest expressions for $R(t)$ that the relationship between L, T_L, and X may not be of a nice simple form. To illustrate this fact, equations (2) and (3) for $R(t) = R$ (a constant), $R(t) = ae^{-bt} + c$, and $R(t) = \frac{f}{g+t} + h$ are presented below. Their derivations are presented in the appendix.

If $R(t) = R$ (a constant) we get for Manning Model No. 1:

$$L = \frac{X R T_L}{1 + \frac{R}{W}(X-1)} \tag{4}$$

For Manning Model No. 2 we get

$$L = T_L W \left[1 - (1 - R/W)^X \right] \tag{5}$$

If $R(t) = ae^{-bt} + c$ then Manning Model No. 1 gives

$$L = \frac{Xa \left[1 - e^{-b\left[T_L - \frac{L}{W}(X-1)\right]}\right] + Xc\, T_L}{b\left(1 + \frac{c}{W}(X-1)\right)} \tag{6}$$

and Manning Model No. 2 does not yield a simple closed form (see the appendix).

If $R(t) = \frac{f}{g+t} + h$ then Manning Model No. 1 gives

$$L = \frac{Xf \ln g^{-1}\left[T_L - \frac{L}{W}\left(\frac{X-1}{X}\right) + g\right] + XhT_L}{1 + \frac{h}{W}(X-1)} \tag{7}$$

nd again Manning Model No. 2 yields no simple closed form.

The two time varying expressions for $R(t)$ we have chosen appear to be representative of the simplest forms that the actual production curve may take when fatigue is involved. $R(t) = R$ (a constant) is included to provide a reference point to which the other expressions may be compared. However, the expression $R(t) = R$ (a constant) may be quite satisfactory for situations where T is small ($T < 1$ hour).

The expression $R(t) = ae^{-bt} + c$ can represent precisely the continuous form of equation (1) if $c = o$. It should be remembered, however, that equation (1) was based on only two hours of continuous data and therefore may not represent reality when $t > 2$. About all that can be said safely is that the average production rate curve, $R(t)$, decreases with time and that it may have a shape similar to an exponential decay curve. Until further data is available it is pointless to present analysis of all the different mathematical expressions which might give similar curve shapes. This paper therefore will be restricted to an analysis involving the constant and two time varying expressions for $R(t)$ given above.

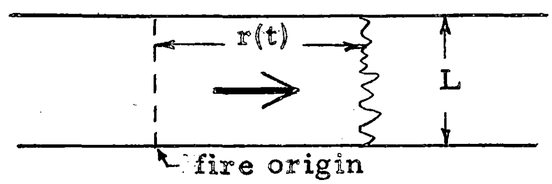
Figure 1 shows representative curves for the three expressions we have assumed for $R(t)$ for one man constructing fire line in a typical fuel, fuel type 1 (grass). The parameters of the two time varying expressions were determined by curve fits to the U. S. Forest Service data (3) over the time interval $0 \leq t \leq 2$ hours for assumed values of $c = o$ and $h = o$. When $c = 80$ the exponential curve approximates quite closely the curve for $R(t) = \frac{f}{g + t} + h$ when $h = 0$ for the time range shown in figure 1. The value for $R(t) = R$ was taken as the "0.833K" term of equation (1).

Figures 2, 3, and 4 indicate the relationship between L, T_L, and X for the three R(t) expression for both manning models. These figures verify that there is no difference between the two manning models when X = 1. The difference between the curves for the manning models for a certain value of X > 1 is due to the fact that more time can be spent in Model No. 2 than in Model No. 1 on actual line construction.

III. Manning Models and Fire Spread

A. Fire-Spread Models. --Let us first consider two simple fire-spread models which might occur in the real world.

The first model is that of a fire spreading on only one front as shown in the sketch.

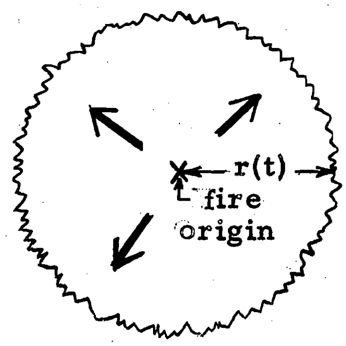


This might be representative of a fire burning through a steep-walled canyon with a control line already completed across the canyon at the point of the fire origin. The width of the fire front is L, the distance across the canyon. The fire is spreading up canyon and its distance from the point of origin at any time is some arbitrary function of time after start t. For example, the fire front might have both velocity and acceleration components and therefore:

$$r(t) = vt + \frac{1}{2} ht^2$$

where v is the linear velocity and h is the linear acceleration.

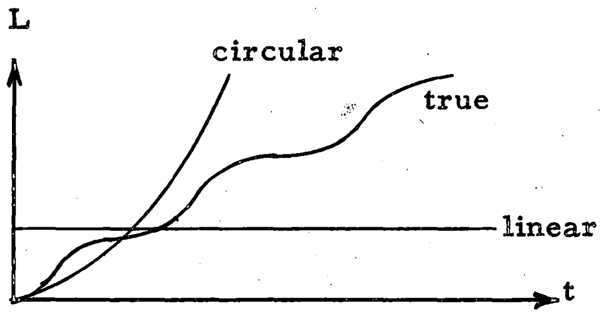
The second spread model is that of a fire spreading equally in all directions outward from the point of origin.



Hence, at any time after start, the fire perimeter describes a circle. The radius of the fire, r(t), is an arbitrary function of time after start t. The length of the fire perimeter is therefore increasing with time.

If we now consider a real fire we know from past history that it will have a perimeter which is increasing with time during some portion of the time period that the fire is uncontrolled. Since the formulation of a fire-spread model representing reality has not yet been developed, the mathematical expression describing the way in which perimeter varies with time is not available. However, under certain conditions it may be possible to predict, at least for short periods of time when the fire is active, the way in which the perimeter is actually increasing.

The curves of length of uncontrolled perimeter as a function of time after the fire starts for the linear, circular, and possible true spread models might look like the following sketch.



If we had such curves available we could then combine them with the equations or curves obtained from the analysis of the manning models to find the minimum number of men that would be required to insure that a complete control line was constructed by a given time after the fire had started. To see how this might be done let us first look at some firefighting tactics we might wish to employ.

B. Control Tactics.--The current firefighting tactics in actual use by crews of men employing hand tools include:

1. Direct method
2. Indirect method
3. Parallel method

The direct method involves constructing control line right on the edge of the fire. From the firefighter's standpoint this approach is the most desirable when conditions permit.

The indirect method involves constructing control line at a considerable distance from the edge of the fire and is normally used for fast moving fires or in situations where unsafe working conditions exist. Backfires are usually set to burn away the fuel between the control line and the edge of the fire.

The parallel method involves constructing line at some short distance from the fire and approximately parallel to its edge. The fuel between the line and the fire may then be burned away to provide additional protection.

Suppose we wish to use the indirect method and therefore begin line construction at some safe distance from the fire's edge. For both ideal spread models and the true fire case let us consider that we wish to construct the necessary length of control line parallel to the fire's edge and at a distance far enough from the fire so that the line can be completed, including backfiring, by the time the fire reaches it. Under the tactic we have chosen the shape and length of the control line will be equivalent to that of the fire's perimeter at the time the fire reaches the line.

For the linear spread model the length of control line needed is L, the canyon's width, and will not change over time. However, since the length of the fire perimeter in the circular spread model and true fire case is increasing with time, we are confronted with the fact that the farther from the fire front we start line construction the longer will be the required control line.

From curves or equations we may be able to determine the length of the fire perimeter at any time. The equations given in the analysis of the manning models give the amount of line that can be constructed by a certain sized crew of men by any given time after attack if R (t) is known. Thus by combining the equations or curves from the manning model analysis with curves or equations describing fire perimeter growth we can determine the minimum size of a crew which will insure control of the fire for various time of attack and times from attack to control.

If we combine curves describing the length of fire perimeter as a function of time with the manning model curves for some time of attack T_A after the fire starts, the resulting figure would look like figure 5. The region above the perimeter curve contains the feasible manning models with their corresponding crew size values which would insure that the needed line will be completed by the time the fire reaches it.

A similar analysis would enable us to also determine the minimum number of men required to construct a portion of a complete control line. Such information would be of interest in fighting a real fire where the procedure might be to assign different crews of men to different segments of the fire perimeter.

IV. Effects of Shifts

If the time required to control a fire or a segment of its perimeter is much longer than twelve hours, then in practice shifts of men are usually introduced to provide relief for tired crews. In addition to providing relief the introduction of these shifts increases the overall production efficiency and consequently the amount of line constructed during a given period of time. In practice each of these shifts usually constructs a segment of line during their work period which takes them from one "tie point" to the next. These tie points are normally chosen as locations where crews may be delivered and picked up by transportation vehicles.

Let us therefore assume that the manning models are valid and that when shifts are introduced that they are delivered to the location where they are to start line construction. We can then derive

expressions for $L_n(X, T_L)$, the total length of line constructed in a time period T_L by n shifts of crews containing X men. The expressions for L_n for general $R(t)$ is given by equation (8) for Manning Model No. 1

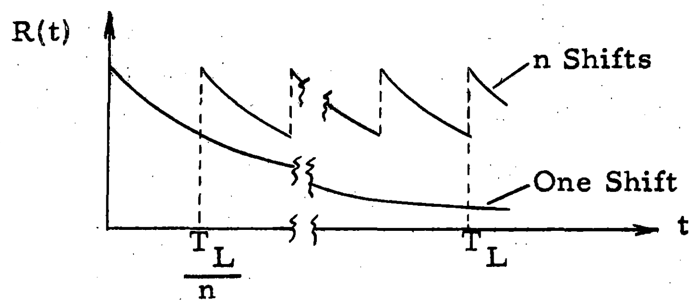
$$L_n = nX \int_0^{\frac{T_L}{n} - \frac{L_n}{nW}\left[\frac{X-1}{X}\right]} R(t)dt \qquad (8)$$

For Manning Model No. 2 the expression is given by

$$L_n = n \sum_{j=1}^{X} \int_0^{\frac{T_L}{n} - \sum_{i=0}^{j-1} \frac{\ell_i}{W}} R(t)dt \qquad \text{where } \ell_0 = 0 \qquad (9)$$

The derivations of equations (8) and (9) and the equation for ℓ_i in equation (9) are given in the appendix. These derivations are based on the assumption that n is a positive integer.

From the expressions for L_n, given in the appendix for the $R(t)$ equations considered in this paper, we also see that if $R(t) = R$ (a constant) then the total length of line produced by n crews in T_L is $L_{n>1} = L_{n=1}$. However, if $R(t)$ corresponds to the time varying expressions given above then $L_{n>1} > L_{n=1}$. The increase in length of line produced for the latter case can be considered to result from a new composite curve for $R(t)$ which gives a higher average $R(t)$ than the curve for $n = 1$ over the time period T_L. The new $R(t)$ curve has a sawtooth form as indicated in the sketch below.



If more line can be constructed by crews of men working under a multiple shift situation than under a single shift situation then it will also be true that there may be various combinations of n and X which would insure that a given length of line L_n is constructed in a given time T_L. To determine the best combination of n and X to use requires the establishing of a decision criterion.

A reasonable criterion might be that of using the combination which gives the minimum cost per unit length of control line constructed. To illustrate how this decision criterion could be used let us assume that the total direct operating cost per unit length of control line produced is represented by:

$$C_L(n) = \frac{C_x X T_L}{L_n} + \frac{n\, C_s}{L_n} \qquad (10)$$

where: C_x = manpower costs per unit time

C_s = transportation costs per crew per shift.

The C_s term could also include other costs such as administration, feeding, equipment preparation, etc., which are incurred when a shift of men is sent onto the line.

If the relationship between X, T_L, L_n, and n for time varying $R(t)$ is of a simple mathematical form we could obtain an expression for optimal (least cost) X as a function of T_L, L_n, n, C_x, and C_s

by setting the derivative with respect to X of equation (10) equal to zero. However, the analysis presented above indicates that for some time varying forms of $R(t)$ no simple relationship will exist between L_n, X, T_L, and n. In such cases we must assume a value for n for a given set of values of T_L and L_n and solve for X using equation (8) or (9). This value for X can then be introduced into equation (10) to determine the value of $C_L(n)$. The combination of n and X which gives the minimum value for $C_L(n)$ will then be the one to use.

As an example let us consider using Manning Model No. 1 with $R(t) = ae^{-bt} + c$ (let $c = 1$) in fuel type 1. If $T_L = 24$ hours and $L_n = 10,000$ feet the minimum value of X for a given n which will insure that L_n is constructed in the time period of length T_L is shown in the following table.

Feasible Combinations		$C_L(n)$, dollars/1000 ft.
n	X	
1	5	2.70
2	4	2.04
3	2	1.86
4	2	2.16
5	1	1.98

The corresponding values of $C_L(n)$ are shown in the right-hand column of this table for assumed values of $C_x = \$2.00$/man-hour and $C_s = \$30$/crew-shift. An examination of the table shows that the minimum cost choice would be $n = 3$ and $X = 2$ or three shifts of two-man crews. Each crew would therefore work an eight hour shift.

In practice the values that C_x can take do not vary by more than ten percent for the various degrees of skill of crew members. The value of C_s, however, is highly variable due to the many different modes of transportation available today. Therefore the value of $C_L(n)$ and consequently the least cost combination of n and X are highly dependent on the value of C_s. For example, if $C_s = \$10$/crew-shift had been used in the calculation of $C_L(n)$ in the above table then the least cost combination would have been $n = 5$ and $X = 1$. If we had used $C_s = \$100$/crew-shift the least cost combination would have been $n = 1$ and $X = 5$.

V. Concluding Remarks

This analysis of the influence of hand crews on general fire growth has attempted to touch briefly upon the major influences. The assumptions made in most cases were either to simplify the mathematical analysis or to fill voids in the quantitative knowledge of the actual activities of hand crews on a fire.

There appears to be no substitute at present for hand crews in firefighting operations. Therefore, a definite effort should be made to use these crews as effectively as possible. The first step which should be taken is to obtain accurate and complete data on hand crew productivity on a fire. This data should include production rates over differing lengths of time under all types of working conditions, in all fuel types and topography.

Future operational studies will examine such topics as other tactical approaches, manning models with holding activities and varying walking rates, differing R(t) curves for men within a crew and between shifts, assignment of multiple crews to the line at the same time, and the effects of probablistic curves of R(t) on tactics and alternatives.

The ultimate goal of these studies will be to determine the optimal allocation of available manpower on a real fire and the best alternatives to choose for changing conditions.

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APPENDIX

I. Formulation of the Manning Models for a Single Shift.

A. Notation

T_L = total time period of a man or crew on duty.

L = total length of control line constructed by a crew during the time period T_L.

X = number of men (or crews) assigned to build the line of length L.

t_j = actual time that the j^{th} man spends constructing control line during the time period T_L.

ℓ_j = length of control line constructed by the j^{th} man during the time period t_j.

w_j = time required for the j^{th} man to walk to and/or from his assigned section of line.

$R(t)$ = average rate of line construction by one man during the t^{th} unit of time after he starts work, (dist./man-time).

W = average walking rate of a man along the control line, (dist./time).

B. Manning Model No. 1--For this model the following relations are characteristic:

$$T_L = w_j + t_j$$

(1) $$L = Ww_j + \ell_j \qquad \text{for all } 1 \leq j \leq X$$

$$\ell_j = L/X$$

1. General $R(t)$--For the general form of $R(t)$ the expression for ℓ_j is given by

(2) $$\ell_j = \int_0^{t_j} R(t)dt$$

Since $\ell_j = L/X = \ell$ for all j then $t_j = t$ for all j and from equation (1)

(3) $$t_j = t = T_L - \frac{L}{W}\left(\frac{X-1}{X}\right)$$

since

(4) $$w_j = w = \frac{L}{W}\left(\frac{X-1}{X}\right) .$$

Therefore equations (1), (2) and (3) can be combined to get

(5) $$L = X\int_0^{T_L - \frac{L}{W}\left(\frac{X-1}{X}\right)} R(t)dt .$$

Equation (5) may be used to solve for one of the three parameters, L , T_L , or X , as a function of the other two if the form of $R(t)$ is known. The results from equation (5) can be substituted back into equations (3) and (4) to get t and w . The value for ℓ can be determined from $\ell = L/X$.

2. $R(t) = R$ (a constant)--For this form of $R(t)$ equation (5) gives

(6) $$L = \frac{XRT_L}{1 + \frac{R}{W}(X-1)}$$

Equation (6) may be rewritten to give

(7) $$T_L = \frac{L}{RX}\left(1 + \frac{R}{W}(X-1)\right)$$

(8) $$X = \frac{L}{R}\left(\frac{W-R}{WT_L - L}\right)$$

3. $R(t) = ae^{-bt} + c$ -- In this case equation (5) gives

(9) $$L = \frac{Xa\left[1-e^{-b\left[T_L - \frac{L}{W}\left(\frac{X-1}{X}\right)\right]}\right] + XcT_L}{b\left[1 + \frac{c}{W}(X-1)\right]}$$

Because of the complexity of equation (9), no simple form can be found for L, T_L, or X as a function of the other two parameters. In an actual problem, solution by Newton's method is the best procedure to use.

4. $R(t) = \frac{f}{g+t} + h$ -- In this case equation (5) gives

(10) $$L = \frac{Xf \ln g^{-1}\left[T_L - \frac{L}{W}\left(\frac{X-1}{X}\right) + g\right] + XhT_L}{1 + \frac{h}{W}(X-1)}$$

The same remarks hold for equation (10) as for equation (9).

C. Manning Model No. 2 -- For this model the following relations are characteristic:

(11) $$T_L = t_1$$

$$T_L = t_j + w_j = t_j + \sum_{i=1}^{j-1} \frac{\ell_i}{W} \quad \text{for} \quad 2 \leq j \leq X$$

$$L = Ww_X + \ell_X$$

where X denotes the last man on the crew to start work.

1. General form of R(t)--Equation (2) applies for the general form of R(t). When $j = 1$, equations (2) and (11) combine to give

(12) $$\ell_1 = \int_0^{T_L} R(t)dt$$

From equation (1) for $j = 2$

(13) $$t_2 = T_L - \frac{\ell_1}{W}$$

and

(14) $$\ell_2 = \int_0^{T_L - \ell_1/W} R(t)dt$$

Therefore for $j = 3$

(15) $$t_3 = T_L - \frac{\ell_1}{W} - \frac{\ell_2}{W}$$

and

(16) $$\ell_3 = \int_0^{T_L - \ell_1/W - \ell_2/W} R(t)dt$$

In general then if we introduce $\ell_0 = 0$ we get

(17) $$\ell_j = \int_0^{T_L - \sum_{i=0}^{j-1} \frac{\ell_i}{W}} R(t)dt$$

Now $L = \sum_{j=1}^{X} \ell_j$ and substitution of equation (17) for ℓ_j gives

(18) $$L = \sum_{j=1}^{X} \int_0^{T_L - \sum_{i=0}^{j-1} \frac{\ell_i}{W}} R(t)dt$$

2. $R(t) = R$ (a constant) -- For this case we get from equation (12)

$$\ell_1 = RT_L$$

then equations (13) and (14) give for $j = 2$

$$t_2 = T_L \left(1 - \frac{R}{W}\right)$$

$$\ell_2 = RT_L \left(1 - \frac{R}{W}\right)$$

and for $j = 3$

$$t_3 = T_L \left(1 - \frac{2R}{W} + \frac{R^2}{W^2}\right) = T_L \left(1 - \frac{R}{W}\right)^2$$

$$\ell_3 = RT_L \left(1 - \frac{R}{W}\right)^2$$

Therefore, in general

(19) $$\ell_j = RT_L \left(1 - \frac{R}{W}\right)^{j-1}$$

Since $L = \sum_{j=1}^{X} \ell_j$ we get

(20) $$L = T_L W \left[1 - (1 - R/W)^X\right]$$

Equation (20) can also be used to get

(21) $$T_L = \frac{L}{W\left[1-(1-R/W)^X\right]}$$

(22) $$X = \ln \left[\frac{L - WT_L}{T_L(R-W)}\right]$$

3. $R(t) = ae^{-bt} + c$ -- For this case equation (12) gives

$$\ell_1 = \frac{a}{b}(1 - e^{-bT_L}) + cT_L$$

For $j = 2$ we get

$$t_2 = T_L - \frac{\ell_1}{W} = T_L \left[1 - \frac{c}{W} - \frac{a}{bWT_L}(1 - e^{-bT_L})\right]$$

Substitution of this expression for t_2 into

$$\ell_2 = \frac{a}{b}(1 - e^{-bt_2}) + ct_2$$

gives a rather complex resulting expression. No simple general closed form for L appears to be obvious for this case. The solution for L for any particular T_L and X values can be obtained by using

$$L = \sum_{j=1}^{X} \ell_j$$

and the appropriate form of equation (17).

4. $R(t) = \frac{f}{g+t} + h$ -- No simple closed form for $L = f(X,T_L)$ can be developed for this form of R(t). The procedure to use for solving for L is the same as that for the preceding form of R(t).

D. General Discussion of the Derivation of the Manning Models.--

An examination of all of the derived equations for L show that if $X = 1$ then these equations are identical for both manning models.

We also note that as the walking rate W becomes very large the length of line constructed by each man in Model No. 2 approaches

$$\ell = \frac{L}{X}$$

which is one of the main properties of Model No. 1. However both models give

$$\lim_{W \to \infty} L = X \int_0^{T_L} R(t)dt$$

indicating that the amount of time each man spends constructing line approaches the total time period T_L as W gets very large.

II. Formulation of the Manning Models for Multiple Shifts.

The following formulation is restricted to the following conditions:

1. Each shift is transported to and starts work at the point where the preceding shift completed its assignment.
2. The number of shifts, n, is a positive integer.

A. Manning Model No. 1--For this model the following relations are characteristic:

$$\frac{T_L}{n} = w_j + t_j$$

(23) $$\frac{L}{n} = Ww_j + \ell_j \qquad \text{for all } 1 \leq j \leq X$$

$$\ell_j = \frac{L}{nX}$$

1. General R(t)--For the general form of R(t) equations (2) and (4) hold. Combining equation (23)and (4) gives

(24) $$T = n^{-1} \left(T_L - \frac{L}{W} \left[\frac{X-1}{X}\right]\right)$$

Equations (2), (23), and (24) then can be combined to get

(25) $$L = nX \int_0^{n^{-1}\left(T_L - \frac{L}{W}\left[\frac{X-1}{X}\right]\right)} R(t)dt$$

2. Specific forms of R(t)--If R(t) = R(a constant) then equation (25) yields

(26) $$L = \frac{XRT_L}{1 + \frac{R}{W}(X-1)}$$

If $R(t) = ae^{-bt} + c$ equation (25) gives

(27) $$L = \frac{\frac{nXa}{b}\left(1-e^{-(b/n)[T_L - L/W(\frac{X-1}{X})]}\right) + XcT_L}{1 + \frac{c}{W}(X-1)}$$

If $R(t) = \frac{f}{g+t} + h$ then equation (25) gives

(28) $$L = \frac{nXf \ln\left[(ng)^{-1}(T_L - \frac{L}{W}(\frac{X-1}{X}) + 1\right] + XhT_L}{1 + \frac{h}{W}(X-1)}$$

B. <u>Manning Model No. 2</u> -- For this model the following relations are characteristic:

$$\frac{T_L}{n} = t_1$$

(29) $$\frac{T_L}{n} = t_j + w_j = t_j + \sum_{i=1}^{j-1} \frac{\ell_j}{W} \qquad \text{for} \quad 2 \leq j \leq X$$

$$\frac{L}{n} = Ww_X + \ell_X$$

where X denotes the last man on the crew to start work.

1. General $R(t)$--Following the analysis for the single shift case, equations (2) and (29) give

(30) $$\ell_j = \int_0^{\frac{T_L}{n} - \sum_{i=0}^{j-1} \frac{\ell_i}{W}} R(t)dt$$

From equation (30) it follows directly that

(31) $$L = n \sum_{j=1}^{X} \int_0^{\frac{T_L}{n} - \sum_{i=0}^{j-1} \frac{\ell_i}{W}} R(t)\,dt \qquad \text{where} \quad \ell_0 = 0 .$$

2. Specific forms of $R(t)$--If $R(t) = R$ (a constant) the following recursion equation is obtained:

(32) $$\ell_j = \frac{RT_L}{n} (1 - R/W)^{j-1}$$

Substitution of equation (32) into $L/n = \sum_{j=1}^{X} \ell_j$ gives

(33) $$L = T_L W[1 - (1 - R/W)^X]$$

A comparison of equations (20) and (33) shows that the n factor does not appear in the multiple shift result when $R(t) = R$ (a constant) and therefore the production of a crew of size X is not influenced by the number of shifts.

When $R(t) = ae^{-bt} + c$ or $R(t) = f/(g + t) + h$ the same difficulties encountered in the single shift case reappear. Solutions of L for specific values of X and T_L can be obtained by using

$$L = n \sum_{j=1}^{X} \ell_j$$

and the appropriate form of equation (30).

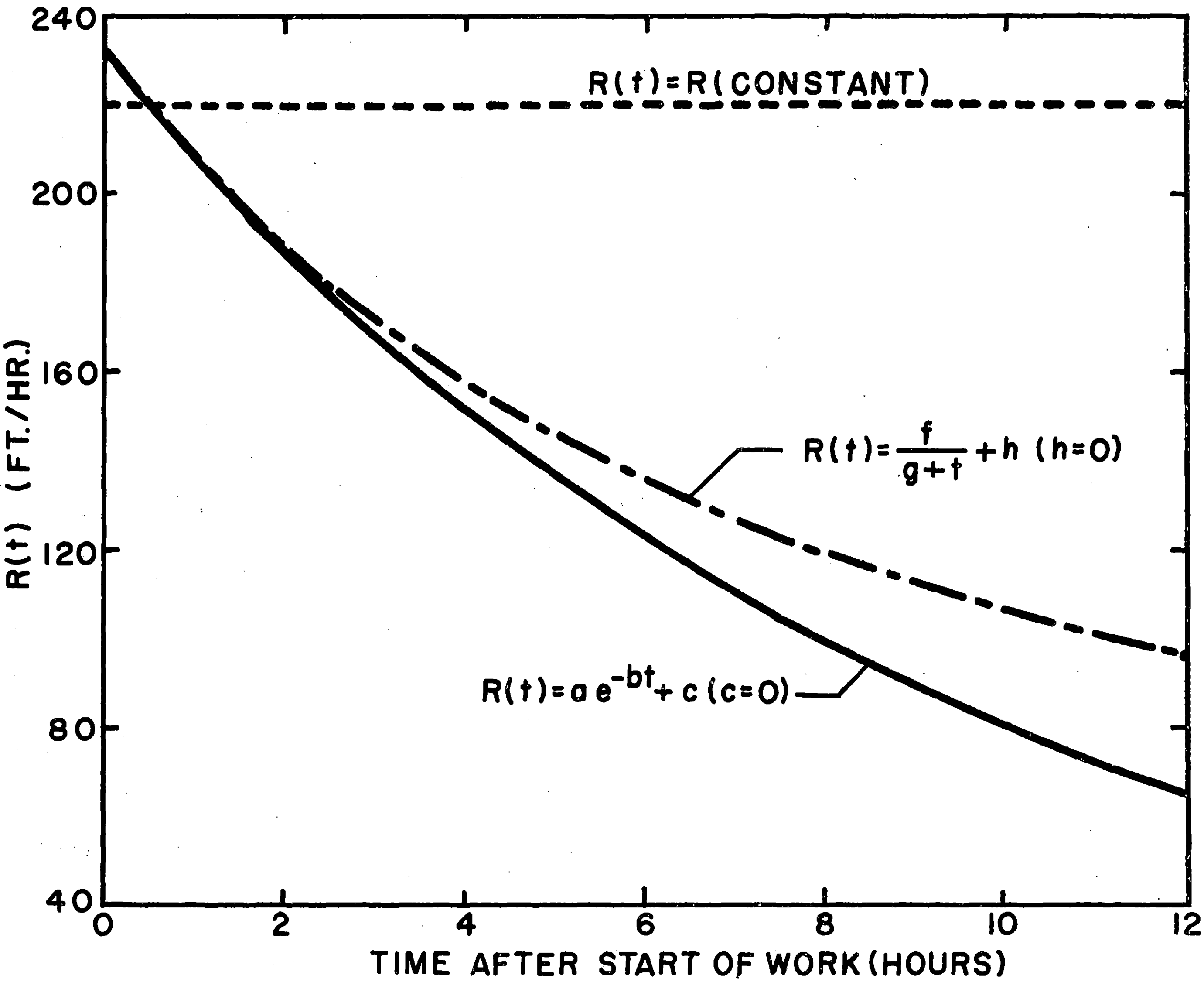

Fig 1.--Curves of average rate of line construction in fuel type 1 (grass).

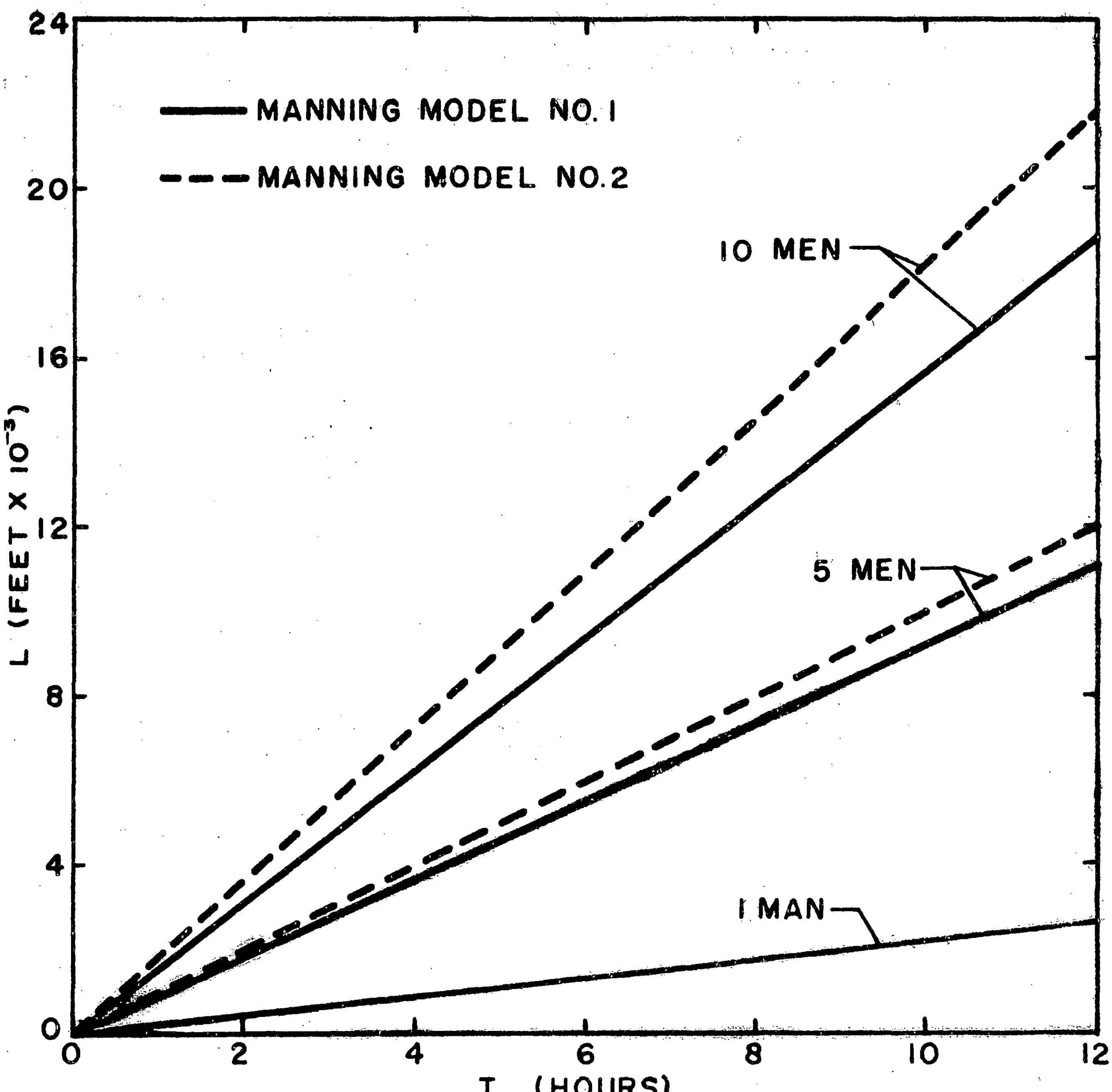

Figure 2.--Amount of line constructed in fuel type 1 as a function of length of time on duty for different sized crews when R(t)=R (a constant).

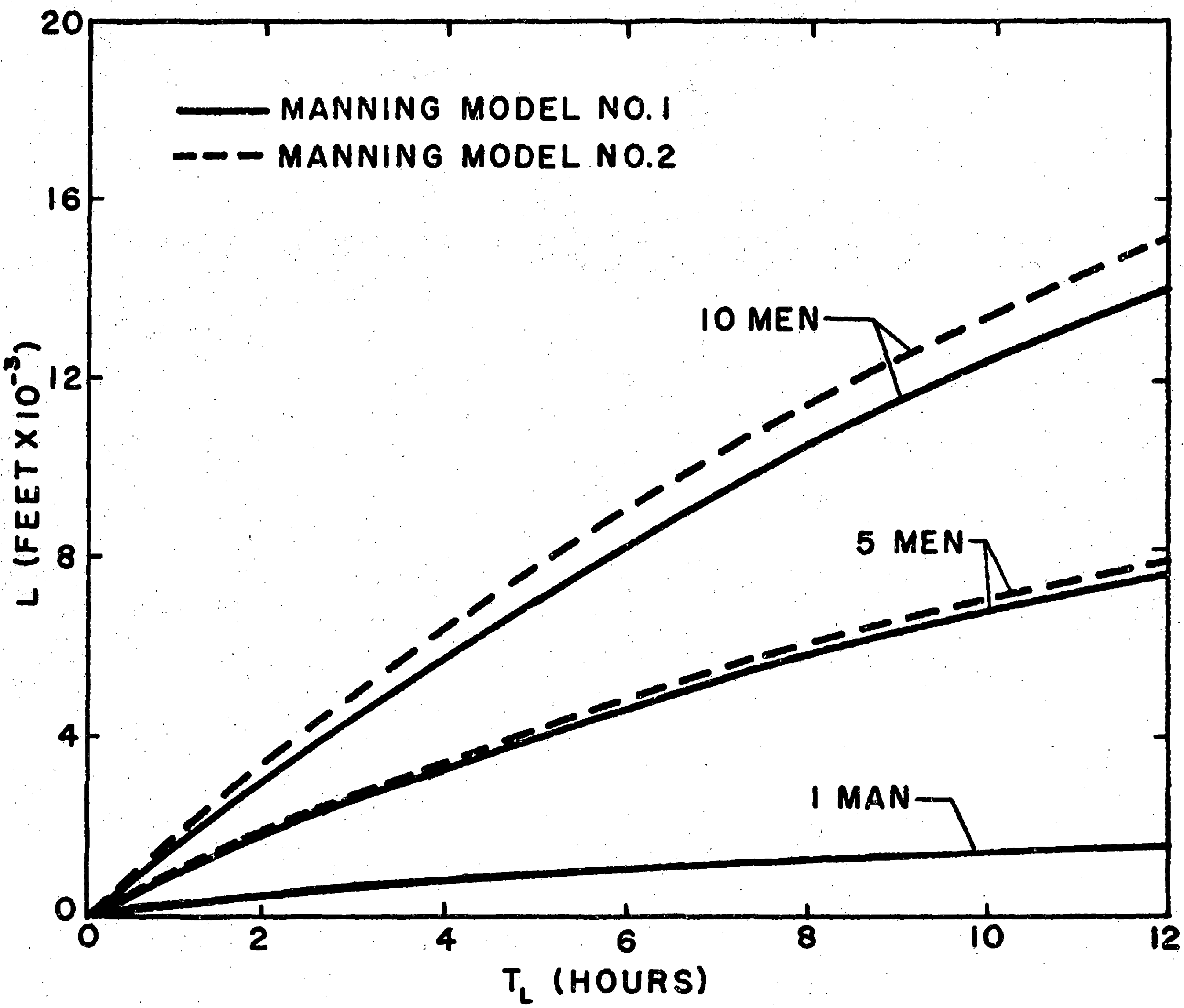

Figure 3.--Amount of line constructed in fuel type 1 as a function of length of time on duty for different sized crews when $R(t) = ae^{-bt} + c$ ($c = 1$).

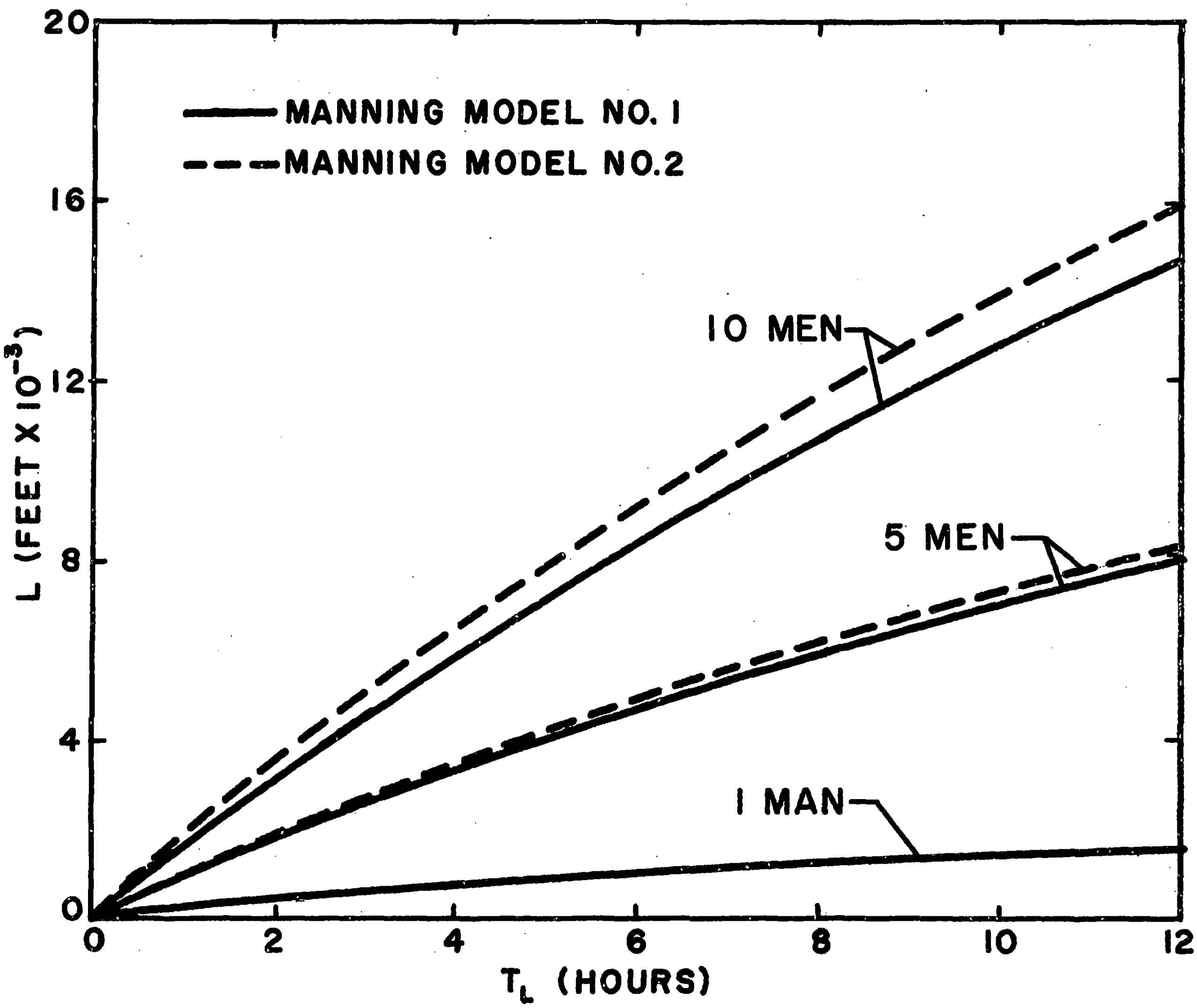

Figure 4.--Amount of line constructed in fuel type 1 as a function of length of time on duty for different sized crews when $R(t) = \frac{f}{g+t} + h$ $(h = 0)$.

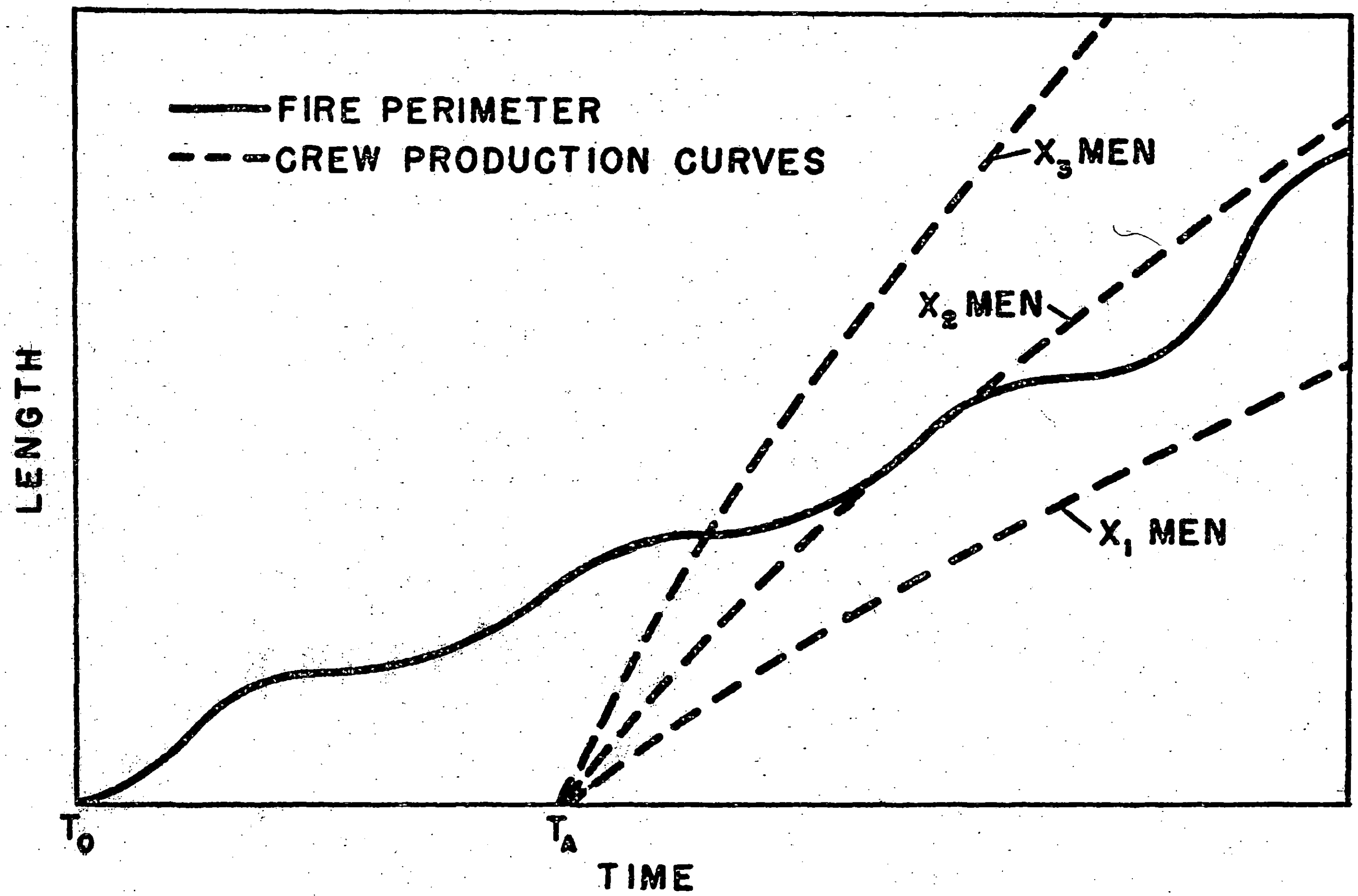

Figure 5.--Illustration of decision curves for determining the minimum crew size to insure control at some given time after attack.